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Is quantum mechanics useful?

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Technologies differ in their explicit utilization of quantum mechanical behaviour. A transistor, despite its roots in energy band structure, does not invoke quantum mechanically coherent transmission between terminals. The impressive progress in the past decade in mesoscopic physics, when combined with studies that have analysed a totally quantum mechanical computational process, suggest that we may be ready to move toward more quantum mechanical procedures for information processing. This paper is a warning signal; this possibility is beset by problems. The case will be made via two separate but complementary arguments. First, by summarizing this author's published comments on computation via totally quantum mechanical coherent Hamiltonians. The computation is likely to suffer from *localization*, i.e. from reflection of the computational trajectory, causing the computation to turn around. Additionally, small errors will accumulate and cause the computation to go off track. This is supplemented by analysis of specific proposals that suggest more detailed machinery than invoked in the general literature on quantum mechanical Hamiltonian computation.

1. Introduction

My short title may be misleading; I am really concerned with utility in the handling of information. Devices and technology vary in their explicit quantum mechanical behaviour. A screw driver seems very classical. Nevertheless its shape, hardness and friction are determined by quantum mechanical interatomic forces. But we do not need to understand those to design, make or use a screw driver. The transistor is a modern device based on the motion of holes and electrons in energy bands. But it really isn't that different from a screw driver; once we know about holes and electrons and mobilities, we do not need to go back to the Schrödinger equation. The overall behaviour of the transistor does not exhibit quantum mechanical coherence; the transistor is not used for Schrödinger cat experiments. The laser with its dependence on quantized energy levels seems more quantum mechanical than the transistor. But even here we have some difficulty finding a clear demarcation when we view a set of oscillators, ranging from an apparently classical oscillator, such as a pendulum clock, to an optically pumped laser. Finally, a Josephson junction, where the relationship between the applied voltage and the emitted frequency involves Planck's constant, is undeniably quantum mechanical. Circuits involving Josephson junctions are invoked in Schrödinger cat experiment proposals. We want to suggest here that information handling techniques, despite the pressure for miniaturization, should not go too far along the kind of chain we have described. We can argue for this in two very different

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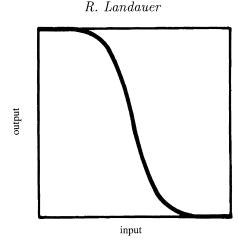


Figure 1. Idealized response of a digital inverting circuit.

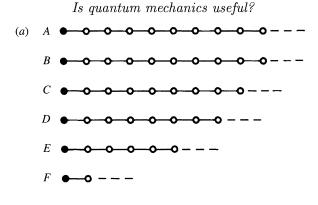
ways. First, by invoking very broad and general considerations prompted by repeated proposals in the literature for totally Hamiltonian computational systems. We then amplify this by considering some much more specific proposals.

In ordinary computer logic the signal is typically restandardized at every stage; i.e., pushed back towards its intended value. Figure 1 from Keyes (1991) illustrates the behaviour of an inverter. Small errors in the input, representing deviations from the intended 0 or 1 signal, are reduced in a properly designed stage. Often, this is accomplished by signals whose swing is limited by a power supply voltage and by ground. This is much like a door, or a wall switch, that can be opened or closed without delicate regard to the exact required force. Thus, as shown in figure 1, for input signals near their desired values, phase space is compressed. This is not characteristic of the unitary time evolution of conservative Hamiltonian systems, but instead requires dissipation. The reduced sensitivity to the exact input, near the ends of the signal range shown in figure 1, requires a compensating gain in the middle of the range; there the output changes more than the input. Again, this is not Hamiltonian behaviour.

A restandardization procedure is needed to prevent the accumulation of successive errors for logic extending over many stages. But this implies throwing away information about the error, and throwing away information is a dissipative process (Bennett 1988). That is the basic problem faced by Hamiltonian computation, whether classical, as in the widely discussed billiard ball model (Fredkin & Toffoli 1982), or quantum mechanical.

2. Hamiltonian computation

Computation in quantum mechanical Hamiltonian systems was originally described by Benioff (1982a, b), and this work has been successfully elaborated by a number of others (Zurek 1984; Peres 1985; Feynman 1986; Landauer 1986; Margolus 1986). In these formulations, computation is viewed as symbolically illustrated in figure 2a. Each line in figure 2a represents a Hamiltonian evolution starting from an initial state at the left, determined by the program loading. We then move to the right through a sequence of 1:1 logical and physical mappings. Each successive circle along the track denotes a computational state. The cited papers show that



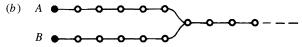


Figure 2. (a) One-to-one computation. The left-hand end of a horizontal chain represents the initial state, and forward computation represents motion to the right, through a sequence of states represented by successive open circles. Different capital letters correspond to different initial states, that is, different programs. (b) Information-discarding junction. Two computational paths, moving to the right, merge into one.

Hamiltonian operators can be specified which supply an interaction between nearby bits so as to cause these to change in time, just as we would want in a computer. At each of the states specified by the circles in figure 2a, each bit is guaranteed to be in a 0 or 1 state; it is not in a quantum-mechanical superposition of these. (But see end of this section for a brief allusion to quantum parallelism.) At the end of the computation the information can be transferred to another apparatus and the computation reversed to its initial state, to reset the computer. Incidentally, this author believes that the reloading process, after reversal, deserves more explicit description than the literature has provided. Information discarding, illustrated in figure 2b, is not essential (Bennett 1988).

What are the difficulties? They are two-fold, but both problems are related to the fact that the Hamiltonian is unlikely to be perfect. There are manufacturing defects, i.e. the Hamiltonian will deviate slightly from its intended form. Furthermore, coupling to the rest of the world will manifest itself as friction and noise. This causes two problems, discussed repeatedly by this author (Landauer 1993, 1991), and we will not repeat these in detail. First of all there is the likelihood, resulting from the irregularities in the Hamiltonian, that the computation will be reflected in its progress along the tracks of figure 2, and turned around prematurely. This is familiar to condensed matter physicists who study electron propagation along chains of the sort shown in figure 2, as localization. Unfortunately, those who have generated papers on the quantum mechanical computational process are not condensed matter physicists, and have simply ignored this question. Localization in computation may not be an uncircumventable problem, but the remedy will not be found unless the problem is admitted. One possible way out will be discussed in §4.

The second problem relates to figure 1, and has already been listed. The restandardization used in figure 1 is not available in a Hamiltonian system. Therefore, errors will pile up, and the computation will go off track. Restandardization requires erasure of the piled-up error. Erasure, in turn (Bennett 1988), requires energy dis-

sipation, and cannot be performed in a Hamiltonian system. Again, there may be ways out, but they will not be identified if the problem is hidden. Rather than repeat and elaborate on my earlier discussion of these problems, in §3 and 4 we turn to an analysis of more specific proposals. These, in contrast to figure 2, try to supply specific device kinetics; they try to explain how the Hamiltonian is realized.

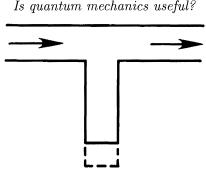
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Finally, let us briefly allude here to quantum parallelism, as advocated in Deutsch (1985, 1989), Jozsa (1991) and Deutsch & Jozsa (1991). In quantum mechanics we face an interference between alternative possible histories. In a two-slit diffraction experiment, the two alternative paths interfere at the detector. Similarly, the alternative computational paths shown in figure 2a can interfere if the initial state is a coherent quantum mechanical superposition of the possible initial states shown in figure 2a. The interference, of course, requires a suitable measurement at the output. Thus, a result can be obtained which depends on all of the simultaneous paths. It is, in effect, a cheap form of parallelism, without enlarging the apparatus or the number of computational steps. The interference, of course, requires a strictly coherent evolution. Advocates of this quantum parallelism readily admit that a totally coherent evolution may be hard to obtain in practice. They do not face the question we have raised; is such a totally coherent evolution desirable?

3. Controlled interference

The continuing motion in computer electronics toward smaller devices, when combined with the rapid progress in mesoscopic physics in the past decade, naturally leads to the expectation that some of the exciting and novel effects in that field will lead to new logic devices. A number of proposals in that direction exist, probably more than this author knows. These proposals are based on single-electron tunnelling, resonant tunnelling, the interaction of quantum dots, and controlled Aharonov–Bohm interference, among others. The merits and deficiencies of these suggestions vary. We shall select one as a prototype, admitting that its faults are not all applicable to all of these proposals.

Perhaps the most basic problem is unrelated to the detailed device kinetics. Smaller devices are more delicate. But as we miniaturize, we try to use more devices. Thus, each device, circuit and connection has to become more reliable, not less. The resulting costs for development, tooling, testing and fabrication have grown explosively with each round of miniaturization. That forces a slowdown, as explained by the Chairman of the Board of Intel (Moore 1993). The cost escalation is a result of miniaturization, not of the details of transistor physics. It would be silly to assume that a transition to new and poorly understood technologies and/or materials would make the cost problems disappear; they would get far worse. Our candidate for detailed analysis is illustrated in figure 3, which shows a horizontal section of electron waveguide with a lateral stub. The stub can be generalized to a resonant cavity coupled to the horizontal section. The waveguide is, most likely, chosen to be only wide enough to allow one transverse mode, or channel, with an energy below that of the Fermi level. The effective electrical length of the stub is controlled through gate voltages. Waves going down into the stub are reflected and interfere, constructively or destructively, with those that have been transmitted directly. Thus, transmission can be controlled through the effective length of the stub and, in principle, can be changed from relatively complete transmission to relatively complete reflection. The list of problems faced by this proposal is very similar to the list faced by controlling



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Figure 3. Transmission through upper horizontal section is controlled through the length of the vertical stub.

transmission through a loop with two attached leads. In the case of the loop, the interference can be controlled through an Aharonov–Bohm flux, or through a gate controlling the potential along one of the two interfering paths.

One of our reasons for choosing the candidate illustrated in figure 3: its problems were already listed in earlier papers (Landauer 1989a, b), but such criticism is rarely reflected in further papers. Landauer (1989a, b) list some of the early proposals related to figure 3. Some very recent studies of the device in figure 3 are given in Porod et al. (1992, 1993), Popov & Popova (1993) and Aihara et al. (1993). Among these, only Aihara et al. (1993) still claim utility.

The proposals for controlling transmission through an attached lead, or cavity, face the following problems beyond that common to all small devices.

- (i) What happens to electrons that are reflected later on and turn around? The published proposals ignore this question.
- (ii) The current flow that is controlled by transmission must in turn control the transmission of a subsequent device by charging a gate. Thus, at the output of the device there must be a voltage swing. How does this affect transmission? The proposals again ignore this point.
- (iii) Small devices carry little current. This is no problem if, as the device and its current are scaled down, the capacitance of the subsequent devices which have to be charged, scale similarly. Unfortunately, interconnection capacitances are unlikely to be that cooperative. In particular, mesoscopic devices tend to have impedances of the order of $13\,000\,\Omega$, very different from the impedances of transmission lines that are typically of the order of $100\,\Omega$.
- (iv) Mesoscopic effects often have to be studied by lock-in amplifiers. Not exactly what you want in a computer circuit. Mesoscopic effects are also, typically, studied at low temperatures, often at mK. Fifteen years ago, in connection with Josephson junction circuitry, we could still argue that in very large systems 4 K was not all that unreasonable. Today, however, in an age of laptops and workstations, that seems to be an unlikely direction.
- (v) Mesoscopic experiments have given rise to some slogans: universal fluctuations and fingerprint of the sample. Mesoscopic samples, closely related to figure 3, have also been studied as examples of quantum chaos (Marcus et al. 1992; Baranger et al. 1993). All of this relates to the fact that quantum mechanical scattering is very sensitive to details (Subramaniam et al. 1990). A small deviation from the exact shape shown in figure 3 matters! And if there are no attempts to restandardize signals by the approach illustrated via figure 1, then the error caused in successive stages will simply accumulate. Mesoscopic samples stand in contrast to conventional

devices, e.g. ordinary resistors, in which we average over the behaviour of a great many independent incoherent subvolumes.

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This author (Landauer 1990) has emphasized that our system is loaded in favour of the uncritical proponents; critical evaluations of the sort given above are rarely cited. Do we need to remind the reader of the many enthusiastic evaluations of logic based on direct interactions of optical signals without the need for electrical interactions, summarized in Landauer (1990). Surely, by now, it is clear that the excessive optimism was not warranted. A critical evaluation (Landauer 1976) was published as early as 1976, but given little attention.

4. Interacting arrays of two-level systems

In this section we will analyse a more sophisticated proposal by Lloyd (1993, 1994) elaborating upon earlier notions by Mahler (Teich et al. 1988; Teich & Mahler 1992; Körner & Mahler 1993), and a still earlier proposal by Haddon & Stillinger (1982). Generalization in a different direction, not evaluated here, has been given by Biafore (1994). A two-level system can be driven from one of its states to the other by a π pulse satisfying

 $\hbar^{-1} \int \mu_{\rm B} \mathcal{E}(t) \, \mathrm{d}t = \pi,$ (4.1)

where $\mu_{\rm B}$ is the dipole moment associated with the transition and \mathcal{E} is the magnitude of the envelope of the component of the field aligned with the dipole moment. The radiative field is assumed to be at the resonant frequency of the two-level system. Now, if the two-level system under consideration is coupled to other nearby twolevel systems with different resonant frequencies, the exact resonant frequency of the system under construction will depend on the state of these adjacent systems. Thus, for example, a transition can be induced by a suitably selected frequency if, and only if, the two neighbouring systems (in a linear array) are both in their excited state. Lloyd (1993, 1994) shows that a suitably selected set of successive π pulses, at different frequencies, can cause the neighbouring 'bits' to interact, and to perform the logic required in a computer. The two-level proposal (Lloyd 1993, 1994) is welcome because it tries to go beyond the considerations described in § 2, based on abstract Hamiltonians. Such specifics help us to focus on the problems discussed in §2.

Lloyd's (1993, 1994) proposal admits that the time evolution may not proceed exactly as desired and that error correction is needed. Lloyd claims that these systems are 'true quantum computers as envisioned by Deutsch (1985, 1989; Jozsa 1991; Deutsch & Jozsa 1992).' But if error correction is needed, this is inevitably dissipative and incoherent, and prevents the quantum parallelism sought by Deutsch, and discussed in $\S 2$.

Lloyd also claims that these two-level systems operate with dissipation required only for error correction. But the validity of that claim depends on the treatment of the continuing sequence of incident π pulses. It is my belief that this radiation is changed sufficiently in its interaction with the system so that it has to be discarded and cannot be reused. In that case, the system is far from one which is minimally dissipative. The alternative case, that the π pulses can be recycled, would need further argument and invention. Note that a computer is a system that has to go through a procedure of indeterminate length. Even small unintended changes, if they occur at each step, are unallowable.

The two-level proposal has one particularly intriguing aspect. The sequence of

incident π pulses times the progress. This is not a computation launched with an initial kinetic energy under a time-independent Hamiltonian. Thus, the problem of unintended reversal of the computation, discussed in §2 may, perhaps, be avoided. The advantages of externally timed computation have been discussed before (Landauer & Büttiker 1985), though in a totally different connection. An earlier proposal for timed quantum-mechanical computation (Landauer 1991) does not eliminate unintended reversals, though possibly it may eliminate the *exponential* diminution of transmission that is characteristic of one-dimensional localization. That quantitative analysis is still missing.

Like all such proposals, the two-level system array faces the question: to what extent does the system really act as it is supposed to do? This includes both defects in the applied Hamiltonian as well as some more intrinsic problems. For example: do the π pulses have the exact prescribed value of $\int \mathcal{E} \, dt$? Do the adjacent and interacting two-level systems have exactly the required spacing? Are there interactions with the environment that provide noise and friction? If dissipation has to be invoked, on occasion for error correction, can it be eliminated at other times? Other possible problems are listed in the Appendix.

5. Coupled quantum-dot cells

he notion that an array of quantum dots might constitute a cellular automaton has been the subject of frequent speculations, and was even the topic for a 1990 Office of Naval Research session in London, Workshop on Applications of Quantum-Coupled Devices to Cellular Automata. Unfortunately, these suggestions, typically, do not provide enough details to permit assessment. A brief critique has been provided in (Landauer 1993).

A series of papers from the University of Notre Dame (Tougaw et al. 1993; Lent et al. 1993a, b; Lent & Tougaw 1993, 1994; Tougaw & Lent 1994) has supplied a proposal that does provide enough detail to permit a reaction. It is, in fact, not a proposal for a cellular automaton but rather for the use of interacting quantum dots to simulate random wired logic, and thus circumvents one of the objections raised in Landauer (1993). What we will, for brevity, call Notre Dame logic uses a basic cell shown in figure 4 (adapted from Lent et al. (1993b)). In a linear array of such cells, interaction between adjacent cells favours their alignment into identically polarized states. Tougaw et al. (1993), Lent et al. (1993a, b), Lent & Tougaw (1993, 1994) and Tougaw & Lent (1994) explain how a number of logic functions can be accomplished through these bistable dots, using the propagation of polarization, established at an input, along configurations which are more complex than a simple uniform chain. Lent et al. (1993b) can be considered to be an elaboration of an earlier more casual suggestion (Bakshi et al. 1991) Bandyopadhyay et al. (1994) gives a closely related approach utilizing spin alignment, instead of polarization.

The Notre Dame logic proposals are welcome, because they are concrete. They lead to two types of questions. The first set deals with the practicality of the implementation. How do we place exactly two electrons on each cell? How is that number controlled and/or restored later on, in the presence of leakage, α particles and cosmic rays? Where is the compensating image charge? Is it worthwhile replacing the ordinary wires in computers by long arrays of interacting dots, with their greater sensitivity to defects?

A second question is more fundamental. There will be places where a polarizable

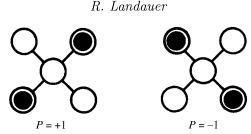


Figure 4. Notre Dame logic cell consists of five coupled quantum dots which are occupied by two electrons. The mutual Coulombic repulsion between the electrons results in bistability between the P=+1 and P=-1 states.

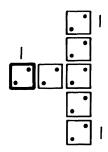


Figure 5. Notre Dame logic fan-out of one signal into two channels. The darker (left-hand) cell has a fixed polarization which constitutes the input.

entity is coupled more effectively to its successors than to its predecessors. Such a case is illustrated in the fan-out shown in figure 5 (adapted from Lent et al. (1993b)). The cell, which has three neighbours, is coupled to two successors, but only one input. Similar problems occur in the inverter and in the wire cross-over schemes proposed in Tougaw et al. (1993), Lent et al. (1993a, b), Lent & Tougaw (1993, 1994) and Tougaw & Lent (1994). In fact, it will occur unintentionally along a 'wire' or uniform linear array of these dots, due to the inevitable random variations in polarization and in coupling strength. An applied signal cannot simply propagate past such a point; the polarizable entity past the weak coupling link will be controlled by an earlier information state. It is, of course, correct, as asserted in Tougaw et al. (1993), Lent et al. (1993a, b), Lent & Tougaw (1993, 1994) and Tougaw & Lent (1994), that the lowest energy state corresponds to that in which the signal has passed the weak link, and continued all the way to the end. But the state in which the signal gets stuck at the weak link is metastable, and thermal fluctuations are needed to get it over a barrier and into the desired state, as recognized in Bandyopadhyay (1993).

We have discussed the quantum dot proposals because they invoke the word quantum and depend on the polarization of quantum states. The proposals, however, need relaxation to the ground state. They are, therefore, inevitably dissipative and do not depend on a quantum mechanically coherent evolution.

Appendix A. Interacting arrays of two-level systems

We list here a number of additional problems faced by this proposal. These difficulties are not necessarily independent of those already listed by Lloyd (1993, 1994).

- (i) In addition to the intentional up and down transitions induced by π pulses, there will also be spontaneous emission.
 - (ii) Does $\mu_{\rm B}$ depend on the state of the neighbouring two-level systems? In that

case, the incident pulse, specified by equation (4.1), has to depend, not only in its frequency, but also in its amplitude, on the desired information evolution (DiVincenzo 1994, personal communication).

- (iii) When neighbours A and C control what happens at B, the transition at B (or lack of it) must also have some influence on the states at A and C.
- (iv) Non-resonant π pulses do not produce an inversion, but do they have no effect at all?
- (v) The possible propagation of an excited state, in an excitonic-like manner, is recognized by Lloyd (1993, 1994). If we have a segment ZABCD, with A and C controlling the transition at B, Z and D must also have some influence, even if only indirectly, via their effect on B and C. Lloyd also recognizes this, and suggests that it can be made the basis of a more complex logic function. But if you push that far enough your elementary logic function simply becomes the mapping of one whole computer state into the next one and that is not really a computer. A computer has to have a design simplicity, whereby the designer can concentrate on the interaction of a few bits at a time. Otherwise it becomes a table look-up machine, where the designer had to anticipate and understand all possible trajectories for the computer.
- (vi) If the array of coupled two-level systems is really evolving coherently, and subject to a single coherent radiation field, why are we allowed to ask what happens at a particular atom? The whole chain of atoms is really one gigantic molecule. This point may be related to the preceding one.

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